NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1306

THREE PAPERS FROM CONFERENCE ON "WING AND TAIL-SURFACE OSCILLATIONS" - MARCH 6-8, 1941, MUNICH

Presented by H. Söhngen, L. Schwarz, and F. Dietze

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I. REMARKS CONCERNING AERODYNAMICALLY BALANCED CONTROL SURFACES*

By H. Söhngen

II. AERODYNAMICALLY EQUIVALENT SYSTEMS FOR VARIOUS FORMS OF CONTROL SURFACES WITHIN THE SCOPE OF THE TWO-DIMENSIONAL WING THEORY**

By L. Schwarz

III. COMPARATIVE CALCULATIONS CONCERNING AERODYNAMIC BALANCE OF CONTROL SURFACES***

By F. Dietze

FOREWORD

Because of the related subject matter in the foregoing papers, which were presented at the conference on "Wing and Tail-Surface Oscillations" at Munich in March 1941, the NACA considered it desirable to combine them into a single Technical Memorandum. These articles follow in the same sequence as originally published in the Lilienthal-Gesellschaft Bericht 135.

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*"Bemerkung zum aerodynamisch innenausgeglichenen Ruder." Lilienthal-Gesellschaft für Luftfahrtforschung Bericht 135, pp. 61-65.

**"Aerodynamische Ersatzsysteme für verschiedene Ruderformen im Rahmen der zweidimensionalen Tragflächentheorie." Lilienthal-Gesellschaft für Luftfahrtforschung Bericht 135, pp. 65-70.

***"Vergleichsrechnungen zum aerodynamischen Ruderinnenausgleich." Lilienthal-Gesellschaft für Luftfahrtforschung Bericht 135, pp. 70-74.

I. REMARKS CONCERNING AERODYNAMICALLY BALANCED CONTROL SURFACES

By H. Söhngen

At the time I announced this lecture, I intended to report on the model with the so-called open gap (fig. 1a) as an idealized model of an aerodynamically-balanced control surface. However, since the formulas for the latter have been published, meanwhile, by Küssner and Schwarz in their report on the oscillating wing with aerodynamically balanced control surface (reference 3), I can limit myself to a few additional remarks.

According to the concept of Küssner and Schwarz, the model with the open gap should be used when the slot is completely filled by the flow. For vanishing flow through the slot, on the other hand, the model with the sealed gap (fig. 1c and fig. 3) should be used. This is quite obvious if the two models are interpreted as flow models. However, one must not disregard the fact that, at the present state of the theory, neither full nor vanishing slot flow can be represented. So far, only such flows can be calculated for which the downwash distribution along a straight line segment is prescribed, or which may be characterized, at least approximately, by such a downwash distribution. In this sense, both models represent specifications for the calculation of the downwash distribution rather than flow models. However, such a downwash distribution alone can characterize neither full nor vanishing slot flow, at least in the case of the vertical step. If one wants to include the influence of the slot on the lift distribution, there exists so far only the possibility of investigating the conditions for steady flow and then transferring them to unsteady flows - of course not without certain questions.

I want to carry out this procedure for the model with the open gap, and to show that the flow through the slot has no noticeable effect on the lift distribution in a linear theory, that, therefore, the initial model and the singly broken plate have, for a constant control-surface deflection, the same lift distribution. This shows simultaneously that the model with the open gap satisfies a very essential condition which must be fulfilled by any model with aerodynamically-balanced control surface, the condition that, for a fixed control-surface deflection, the aerodynamically-balanced and the unbalanced control surface may differ only in the choice of the moment reference point.

Model with the Open Gap

In order to prove the above assertion, we start out from the symmetrical double wing with a finite slot in the direction of the wing chord. For this double wing, it is known (references 1 and 2) that for any finite slot the vortex distribution within the scope of a linear theory (to which we always limit ourselves) is independent of the position of the control-surface axis of rotation. This applies even in case the vortex distribution is actually placed on the mean camber line. Thus the models in figures 2a and 2b have, in the sense of a linear theory, the same lift distribution although they are different with respect to flow. Since this "equivalence" is valid for any finite slot, the limiting process "slot approaching zero" may be performed; we understand by slot here only the slot in direction of the chord. In this limiting process, the model (2b) converges toward the singly broken plate and the lift distribution, as can be easily shown, toward the lift distribution of that plate. In contrast, the model (2a) converges toward the initial model with the open gaps, and the latter therefore has, in a linear theory, the same lift distribution as the broken plate. Thus the slot influence makes itself felt only in these terms of higher order.

This fact becomes understandable if one considers table 1 in which the through-flow quantities for several control-surface chord ratios and rearward positions of the control-surface axis of rotation are compiled. These values, kindly put at my disposal by Miss Ginzel, were calculated according to the theory developed by Flügge-Lotz and Ginzel (reference 4). This numerical table shows the mean through-flow velocities $V/d\eta$ to be of the order of magnitude η, since $\eta = 0.17$. This may be assumed to be the physical reason for the statement, proved above in general, that the flow through the slot does not make itself felt in a linear theory.

Table 1

Mean velocity in the slot of the model (1a) for angle of attack and control-surface angle of 10°

l_R/l	d/l_R	$V/d\eta v$
0.25	0.5	0.18
.35	.46	.20
.35	.39	.18

l = wing chord, l_R = control-surface chord, d = rearward position of the control-surface axis of rotation, η = control-surface angle, v = free-stream velocity, V = through-flow quantity in unit time.

To sum up, one may say that the model with the open gap may be used as the idealized model of an aerodynamically-balanced control surface even when it is required that no flow should pass between fin and control surface. Probably the slot influence, in case of control-surface oscillations, is at least as slight, if not even slighter.

Model with Vertical Step

Finally, I should like to show that the model with vertical step introduced by Küssner and Schwarz leads, for steady conditions, to the same lift distribution as the model with the open gap if Küssner's and Schwarz' calculating procedure is carried out just a little more "exactly." Küssner and Schwarz, it is true, obtain considerably deviating results. I shall show the reason; however, I want to emphasize beforehand that I altogether reject their calculating procedure, including the one carried out more "exactly." I shall discuss my reasons later.

The model with the vertical step represents in Küssner's and Schwarz' report the limiting case of the model with the oblique step. (Compare fig. 3.)

The latter model leads to the model with the vertical step (fig. 1c) by the limiting process $x_1 \longrightarrow x_0$.

The calculating procedure of Küssner and Schwarz is based on Birnbaum's method. One can apply the latter only if the downwash distribution is known. Obviously, there results for the latter

$$w(x) = \begin{cases} 0 & \text{for} \quad -1 < x < x_1 \\ -v \sin \gamma & \quad x_1 < x < x_0 \\ v \sin \eta & \quad x_0 < x < 1 \end{cases}$$

where

$$\gamma = \operatorname{arc} \tan \left(\frac{x_R - x_0}{x_0 - x_1} \tan \eta \right)$$

We split up this downwash distribution in the form

$$w = w_1 + w_2$$

with

$$w_1 = \begin{cases} 0 & (-1 < x < x_1) \\ -v \sin \gamma & (x_1 < x < 1) \end{cases}$$

and

$$w_2 = \begin{cases} 0 & (-1 < x < x_0) \\ v\ (\sin\gamma + \sin\eta) & (x_0 < x < 1) \end{cases}$$

This division precisely corresponds to the division of the model with the oblique step into two singly broken plates made by Küssner and Schwarz. Furthermore we put, as is customary, $x = -\cos\Theta$, $x_0 = -\cos\varphi$, and $x_1 = -\cos\left(\varphi - \delta_R\right)$. If one now takes into consideration that the vortex distribution appertaining to the downwash distribution w_1 (reference 3) is

$$\Gamma_1(x) = -\frac{v}{\pi}\sin\gamma\left\{2(\pi - \varphi + \delta_R)\cot\frac{\Theta}{2} + \ln\frac{1 - \cos(\Theta + \varphi - \delta_R)}{1 - \cos(\Theta - \varphi + \delta_R)}\right\}$$

there results for the model with the oblique step by superposition the vortex distribution

$$\begin{aligned}\Gamma_{\delta_R}(x) = \frac{1}{\pi} v \sin\eta\Bigg\{2(\pi - \varphi)\cot\frac{\Theta}{2} + \\ \ln\frac{1 - \cos(\Theta + \varphi)}{1 - \cos(\Theta - \varphi)}\Bigg\} + \frac{1}{\pi} v \sin\gamma\Bigg\{- 2\delta_R\cot\frac{\Theta}{2} + \\ \ln\frac{\left[1 - \cos(\Theta + \varphi)\right]\left[1 - \cos(\Theta - \varphi + \delta_R)\right]}{\left[1 - \cos(\Theta - \varphi)\right]\left[1 - \cos(\Theta + \varphi - \delta_R)\right]}\Bigg\}\end{aligned}$$

for which the limiting process $\delta_R \longrightarrow 0$ now may be performed without any difficulty. Since the last constituent converges toward zero, there results, therefore, as lift distribution for the model with the vertical step, that of the singly broken plate, therewith the same distribution as for the model with the open gap.

How then is it possible that Küssner and Schwarz arrive at an entirely different result? The reason is that Küssner and Schwarz, at the outset, apply a theory linearized for small flow angles. Thus, in particular, they put $\sin\gamma \sim \gamma$ and $\gamma \sim \dfrac{x_R - x_0}{x_0 - x_1}\eta$ - a simplification which for $x_1 \longrightarrow x_0$ evidently is no longer permissible. Due to this simplification, the last term in the last equation still makes a contribution although one of the factors vanishes.

Although the calculating procedure carried out here a little more exactly did yield a reasonable result, it must still be rejected since the suppositions for which Birnbaum's theory is applicable are not satisfied. This method is exact when the downwash distribution along a straight line segment is prescribed. For the approximate calculation of mean camber lines, it may be used only when these lines deviate only slightly from the straight line and when their curvature is inconsiderable; for only then normals to the mean camber line and to the straight line have approximately the same direction. Obviously this condition is, in a very essential point, not fulfilled on the model with the vertical step.

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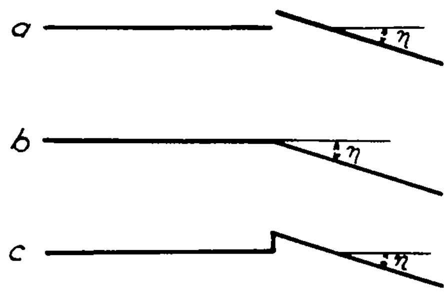

Figure 1.- Control-surface models. a and c - Control surface with aerodynamic balance. b - Control surface without aerodynamic balance. a and b have, for constant control-surface deflection, the same lift distribution.

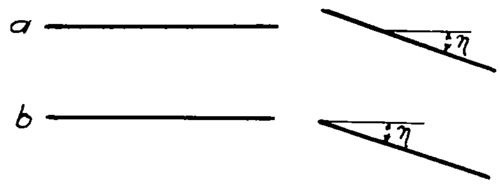

Figure 2.- Two double wings with equal lift distribution.

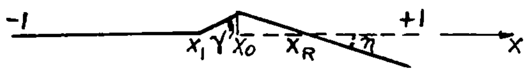

Figure 3.- Control-surface model with oblique step.

Discussion Following the Lecture of H. Söhngen

Küssner: Söhngen himself mentioned in his lecture that his derivations are not unobjectionable. The assumption of a step angle $\gamma = 90^\circ$ contains a contradiction to the presuppositions of the linear theory. This angle must absolutely be kept small at first. In my first report concerning this problem (1937) and likewise in my second (1940), I therefore assumed an arbitrary small step angle. Only in Schwarz' report (Conference on Oscillation 1938) the angle $\gamma = 90^\circ$ has been temporarily introduced as a limiting value, but only for subsequent simplification of an exact result for a practical purpose. It is absolutely imperative that in the derivation the slopes of the mean camber line always remain (infinitely) small, otherwise the linearized theory cannot be carried out free from contradiction.

Addition at the time of the printing:

Altogether, one must keep all perturbation velocities small in order to comply with the suppositions of the theory which would require, among other conditions, also a smooth entrance of the leading edges of fins and control surfaces into the flow. However, that is not done in most cases, and thence result the contradictions mentioned in the first part of the lecture. How an exact result obtained under adherence to the enumerated presuppositions may be subsequently simplified for practical application is another problem requiring separate treatment.

Söhngen:[1] Since the limiting process $\delta_R \longrightarrow 0$ or $\gamma \longrightarrow 90^\circ$, respectively, contradicts the presuppositions of Birnbaum's theory, I rejected the results obtained in this manner. In my opinion, this is an absolutely unobjectionable method. Küssner and Schwarz, on the other hand, did not draw this conclusion. If Küssner says that he or Schwarz performed the limiting process "only for the subsequent simplification of an exact result for a practical purpose," the question still remains open whether such a simplification is permissible. It is not immediately evident since the limiting process leads to the model with the vertical step for which $\sin \gamma$ and $\tan \gamma$ obviously can no longer be replaced by γ. The motivation given by Küssner and Schwarz in the last report (1940) where this limiting process is performed again, namely that δ_R is small and that limiting values exist for the separate coefficients (except for the lift of the control surface) invites objections. In order to make this quite clear, I gave, in the second part of my lecture, the lift distribution of the oblique step in a form for which even all coefficients

[1]Editor's note: Mr. Söhngen wrote this summary after becoming acquainted with Mr. Küssner's altered contribution to the discussion.

with $\delta_R \longrightarrow 0$ tend toward limiting values so that the motivation as given by Küssner and Schwarz could be used here also. However, the coefficients obtained in this manner differ quite considerably from Kussner's and Schwarz' results, although the lift distributions used in both cases (different only in that I did not make use of the simplification - not permissible for large step angles - $\sin \gamma \sim \gamma$, $\tan \gamma \sim \gamma$) are perfectly equivalent for small step angles. The limiting process $\delta_R \longrightarrow 0$ is, in my opinion, justifiable only by either showing that even the perfectly exact calculation (for instance, by means of conformal mapping) leads to the same values as Küssner's and Schwarz' method, or else by showing that the values obtained by the limiting process deviate by a sufficiently small amount from those for small γ's. In the first case, the model with the vertical step would be the flow model, whereas, in the second case, it would represent only a sort of calculation model; however, to my knowledge, such an investigation has not been undertaken so far so that the results obtained by means of the limiting process at the very least must be regarded as not proved.

Concerning the influence of the infinitely large disturbance velocities at the edges, my opinion essentially differs from Küssner's. It is true that in the purely intuitive derivation of Birnbaum's theory, the disturbance velocities are presupposed to be small; however, for profiles consisting of a series of straight lines, this limitation is unimportant since it can easily be demonstrated that a theory which is linear for small angles leads to the same results as Birnbaum's theory. Thus, for instance, Keune's perfectly exact calculation (Jahrbuch 1937 der deutschen Luftfahrtforschung) for the singly broken plate yields the same results as Glauert's calculation which is based on Birnbaum's theory. Contradictions in the results of the first part of my lecture, as a consequence of the infinitely large disturbance velocities prevailing at the edges, could, therefore, not possibly have occurred and did not occur. Only certain deviations from Küssner's and Schwarz' results appeared.

Küssner: Doubts regarding the legitimacy of the sealed gap have been uttered. Instead of discussing reasons or counterreasons, let us calculate a numerical example.

Assume that on a wing with aerodynamically-balanced control surface, the total wing chord extends from -1 to 1, that the leading edge of the control surface lies at 0.40, the control-surface axis at 0.60, and that the slot width is 0.02, that is, 1 percent of the wing chord (compare fig. 3b in the Küssner-Schwarz report henceforth quoted as KS).

Thus

$$
\begin{aligned}
-\cos \varphi &= 0.40 \\
-\cos \chi_R &= 0.60 \\
-\cos(\varphi - \delta_R) &= 0.38
\end{aligned}
\tag{1}
$$

γ is to denote the angle between the oblique line bridging the slot and the x-axis.

In order to make the linearized wing theory applicable, displacement from the x-axis and slope must be small. This means for us in this case that γ must be small, about

$$\gamma \leqq 6^{\circ} \tag{2}$$

Hence follows, due to the known relation between height of the step and control-surface amplitude,

$$\frac{D}{\cos \varphi - \cos \chi_R} = C = \frac{\cos(\varphi - \delta_R) - \cos \varphi}{\cos \varphi - \cos \chi_R} \gamma \tag{3}$$

From equation (2) then follows

$$C \leqq 36' \tag{4}$$

This magnitude of the control-surface amplitude we deem still sufficient to start the flutter.

In any flutter calculation one has to investigate only the start of flutter, since by avoiding the start of flutter, the entire process can be avoided. If one assumes 10° instead of 6° as the limit for the slope of the mean camber line

$$\gamma \leqq 10^{\circ} \tag{2'}$$

C becomes

$$C \leqq 1^{\circ} \tag{4'}$$

Our mean camber line is composed as usual of the singly broken plate and the closed step. We limit ourselves here to the closed step. (Compare fig. 3b in KS.) First, we regard the closed step as a doubly broken plate composed of two singly broken plates. We then obtain, for instance, for the coefficient $k_{\bar{d}}$

$$\pi k_{\bar{d}} = \frac{1}{-0.02}\left\{(2.10599 + 0.89096\omega)(1 + T) + 0.82951\omega + 0.21240\omega^2 - (2.07579 + 0.84914\omega)(1 + T) - 0.79267\omega - 0.19618\omega^2\right\}$$

$$= (1.510 + 2.091\omega)(1 + T) + 1.842\omega + 0.811\omega^2 \qquad (5)$$

This coefficient is, therefore, a difference quotient of the corresponding coefficient of the degree of freedom c, taken at the points 0.40 and 0.38. If we replace this difference quotient by the differential quotient at the point 0.40, we have before us the method we denoted in our common report as closed (vertical) step method. We obtain for the coefficient $k_{\bar{d}}$ according to formula (26) in KS

$$\pi k_{\bar{d}} = (1.528 + 2.076\omega)(1 + T) + 1.833\omega + 0.793\omega^2 \qquad (6)$$

There we suggested replacing the exact coefficient according to formula (5) by the value according to formula (6). We should like to make here a new suggestion as to how to improve the agreement considerably by a slight modification of our method: If we insert in formula (26) of KS not the leading edge of the control surface 0.40, but the center of the slot $-\cos\varphi_m = 0.39$, generally

$$-\cos\varphi_m = -\frac{1}{2}\left[\cos\varphi + \cos(\varphi - \delta)\right] \qquad (7)$$

we obtain for $k_{\bar{d}}$ the value

$$\pi k_{\bar{d}} = (1.510 + 2.091\omega)(1 + T) + 1.842\omega + 0.811\omega^2 \qquad (8)$$

thus within the scope of our accuracy the same value we also obtained in the exact procedure by superposition of two plates in formula (5). More exact calculation shows differences of the order of magnitude δ^2 between equations (5) and (8).

For the steady contribution of the control-surface lift, we provide a special regulation. In KS there was

$$\pi^2 r_{\bar{d}}^* = 2 \ln \tau_{sR} + \Phi_{21}(\varphi)$$

$$= 2 \ln \tau_{sR} - 2(\cos\varphi + \ln \sin^2\varphi)$$

$$= 2 \ln \tau_{sR} + 1.14871 \qquad (9)$$

The exact calculation as doubly broken plate yields

$$\pi^2 r_{\bar{d}}^{*} = 2 \ln \tau_{sR} + 2 \sin \varphi \frac{\sin \varphi - \sin(\varphi - \delta)}{\cos \varphi - \cos(\varphi - \delta)} - 2 \ln \sin^2 \left(\varphi - \frac{\delta}{2}\right)$$

$$= 2 \ln \tau_{sR} + 1.10644 \qquad (10)$$

Instead we newly suggest

$$\pi^2 r_{\bar{d}}^{*} = 2 \ln \tau_{sR} - 2 \left[\sin \varphi \cot \varphi_m + \ln \sin^2 \varphi_m\right]$$

$$= 2 \ln \tau_{sR} + 1.10635 \qquad (11)$$

which sufficiently accurately agrees with equation (10).

This example may show that the closed step represents a perfectly usable instrument of air-force calculation. We note once more explicitly that in our work γ, the angle of the step, nowhere becomes 90°, that in this example γ was selected even $\leq$ 6° or 10°, respectively, and that, therefore, the designation "vertical step" signified only a brief catch word for a purely formal-mathematical method which is to be explained by the customary exaggerated representation of the oscillation deflections in schematic figures.

The comparison with measurements performed in 1938 by the second-named author shows that, for smaller, graphically negligible step width, the equivalent system correctly renders an essential characteristic of the balanced control surface, namely, the very pronounced pressure rise toward the leading edge of the control surface which is known to be accordingly less pronounced in equivalent systems with a break or with open gap. The freely selectable parameter step width permits the adaptation of the steady control-surface moment about the control-surface axis to an experimental value - a measure which seems very desirable according to the expositions of this conference.

At this opportunity, we once more draw attention to a misprint in our report. In Luftfahrtforschung vol. 17, 1940, page 348, top, should read $-\Phi_{18}$ instead of Φ_{18}. Mr. Dietze kindly pointed out to us this error.

Quessel: The first calculations with aerodynamic balance, the results of which have been given in Rühl's lecture at the Hamburg Conference on Oscillation, 1938, were performed with air forces determined at the instigation of Küssner in the following manner: In the determination of the air-force moments acting on the aileron, we extended the integration beyond the control-surface axis A in the direction of the leading edge of the wing to a point B, whereas the profile, as customary, was considered as a singly broken plate with the break in the control-surface axis. We did not deem it at all important that the point B should coincide with the leading edge of the control surface; we rather regard the straight line segment A-B as a parameter independent of the shape of the aileron which is to be determined in such a manner that one obtains for steady conditions the same moment about the control-surface axis, due to a control-surface deflection, as for the actual control surface. In this manner, a parameter which is very significant for the process of flutter, namely, the air damping, is assimilated to actual conditions which is more important than the imitation of the geometrical relations. The calculation mentioned dealt with a qualitative investigation of the influence of the aerodynamic balance; therefore, we assumed perfect aerodynamic balance and determined the straight-line segment A-B so that the steady moment of the air forces about the control-surface axis due to a control-surface deflection disappeared. In other air-force theories as well, a suitable parameter must always be left open for approach to the relations under steady conditions if they are to be applied to flutter calculations.

In his comparison of the calculation results according to the various air-force theories, Dietze started out from the geometrical relations and obtained considerable differences. Had Dietze correlated the air forces of the different theories according to their steady components, that is, had he determined for each theory a suitable parameter so that all theories would, for steady conditions, result in the same moment about the control-surface axis due to control-surface deflection, the differences in the calculation results according to the various theories would not be large. Then the industry should not have to pay any heed to the separate air-force theories which would be of great advantage.

<u>Schwarz</u>: In the case I reported on, a continuous mean camber line was presupposed. The theory can be transferred also to discontinuous mean camber lines as I have already mentioned in my lecture.

<u>Stender</u>: One of the figures showed that the substitution mean camber lines had another nose chord, obviously to attain moment coefficients for control-surface angles which correspond to those of the actual profile form; however, the influence of the angle of attack is more important. The substitution nose chord ought to be selected so that approximately neutrality of the control surface with respect to the angle of attack prevails.

<u>Voigt</u>: Concerning the remarks by Stender: In wind-tunnel tests, we arrived at different results. According to them only the aerodynamic balance compared to the balance of control surfaces is significant.

Dietze: Concerning Quessel's expositions:

1. A comparison between the separate concepts in such a manner that, for equal control-surface chord ratio and equal position of the control-surface axis of rotation, an equal degree of balance also exists at the control surface cannot be made because of the very different emphasis on the balancing effect in the separate concepts and because of the limited variability of the balancing effect by the parameters still at disposal. For instance, in the concept with breaking edge shifted rearward, the same degree of balance as for the concept with open gap can be brought about only when the control-surface leading edge is made the breaking edge; then, however, one can no longer speak of a concept with breaking edge shifted rearward. Furthermore, for the values for control-surface chord ratio and position of the control-surface axis of rotation presented here, in no case (that is, by no suitable selection of the step width) may the same balancing effect be obtained by the concept with oblique step as for the concept with open gap. (That, however, such an agreement might after all be possible occasionally, for other chord ratios and positions of the axis of rotation, is shown, for instance, in fig. 4.)

For average degrees of balance, it will usually be possible to choose between two, more rarely between three concepts. For higher degrees of balance, the concepts with open gap and with closed vertical step are usually the only possibilities left.

2. The plotting of the control-surface moment against the reduced frequency (compare fig. 3) demonstrates that merely the part which is in phase with the motion shows essential differences in the application of the various concepts, and that those differences are almost independent of the frequency. For equal steady degree of balance, the moment curves of the different concepts would almost coincide. In that case, the separate concepts probably will not differ in practice, either, in their effect on the additionally occurring air-force terms (total force, total moment, etc.) and on the critical velocity.

Translated by Mary L. Mahler
National Advisory Committee
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II: AERODYNAMICALLY EQUIVALENT SYSTEMS FOR VARIOUS FORMS OF CONTROL SURFACES WITHIN THE SCOPE OF THE TWO-DIMENSIONAL WING THEORY

By L. Schwarz

Abstract

After a survey of the present state of the theory of the oscillating wing, the equivalent systems so far suggested for wings and control surfaces without and with aerodynamic balance are discussed. Following, it is shown how such equivalent systems may be constructed from experimentally obtained measurements of unsteady pressure distributions. Finally, a method of calculating unsteady pressure distributions from suitable steady pressure-distribution measurements is indicated.

1. Summary of the Theory

The two-dimensional problem of the oscillating wing has come to some sort of close at present. The problem consists in determining the pressure distribution occurring under the influence of the motion of the fluid at the wing surface, in case of a prescribed harmonic deformation of an infinitely thin wing. Under the known assumptions which permit a linearization of this aerodynamic problem, the solution is as follows:

In the customary notation, let

ρ = the air density

v = the free-stream velocity

ν = the angular frequency

ω = the (imaginary) reduced frequency

x = the coordinate in direction of the wing chord

t = the time

z = the harmonic deformation

w = the downwash

Π = the pressure distribution

The downwash then is

$$w = \frac{\partial z}{\partial t} + v\,\frac{\partial z}{\partial x}$$

and the pressure distribution

$$\Pi(x,t) = \frac{2}{\pi}\,\rho v \oint_{-1}^{1} w(\xi,t)\left[\omega\Lambda(x,\xi) + \sqrt{\frac{1-x}{1+x}}\,\sqrt{\frac{1+\xi}{1-\xi}}\left\{\frac{1}{\xi - x} - \frac{1 - T(-i\omega)}{2}\right\}\right]d\xi$$

with

$$\Lambda(x,\xi) = \frac{1}{2}\ln\frac{1 - \xi x + \sqrt{1-\xi^2}\,\sqrt{1-x^2}}{1 - \xi x - \sqrt{1-\xi^2}\,\sqrt{1-x^2}}$$

and

$$T(-i\omega) = \frac{-iH_0^{(2)}(-i\omega) + H_1^{(2)}(-i\omega)}{iH_0^{(2)}(-i\omega) + H_1^{(2)}(-i\omega)}$$

that is, nothing else is required now for determining the pressure distribution from the deformation but integrations, and since forces and moments also may be derived from the pressure distribution by integrations, all aerodynamically interesting quantities may be found by integration.

In every application of the theory, there arises the basic problem of what mean camber line to substitute for the prescribed wing profile. The discussion of this question is in full swing. Following, the mean camber lines or, as we will call them conforming to the custom in other parts of mechanics, the equivalent systems treated so far will be compared. The discussion of basic problems may be limited to the system wing - control-surface.

2. Equivalent Systems for Wings with Control Surfaces and Auxiliary Control Surfaces without Balance

In this case, one chooses as mean camber line of the profile the broken line. If, moreover, an auxiliary control surface is present, one has instead the doubly broken line.

Within the scope of linear theory, it does not signify a limitation that the mean camber line consists of series of straight-lines even though the center line of the profile, for instance a wing with S-shaped camber may be by no means straight. If the S-cambered line in figure 1 performs oscillations, this motion may be composed of the S line at rest and the oscillations of the chord. We need not be concerned with the first portion which is steady, and the treatment of broken lines is generally known.

A sharp break in the mean camber line is the cause of a singularity of logarithmic character in the pressure distribution. It is desirable to avoid such infinite pressure points in theory, since infinite pressures cannot occur in reality. Thus one has to find reasons for slightly altering the mean camber line. A few simple appropriate deliberations may be mentioned.

Rounded-off break.- If the boundary layer is visualized as a layer actually laid around the profile while outside the fluid flows in a potential flow, the outer bounding of the boundary layer represents the profile as compared to the potential flow "outside." Obviously, the corner of the original profile is, therefore, "padded" by the boundary layer. The boundary layer thus has for the broken line the effect of the sharp break being rounded.

We take (compare fig. 2) for the rounding a parabolic arc which is tangent to the two sections of the broken line. We assume the break at a, the beginning of the rounding parabolic arc at $a - f$. It then adjoins for $x = a + f$ the deflected part of the straight line, so that the width of the rounding is $2f$. The deflection of the bent part of the straight line is assumed to be C. The appertaining pressure distribution is to be determined.

The calculation will be formulated so that we may further utilize it later on. If pressure distributions for new mean camber lines are calculated, these camber lines should not only be appropriate for the respective purpose, but should also fit into a reasonable systematization of the possible mean camber lines. We introduce the following mean camber lines:

$$F_r(a;\ x) = \begin{cases} 0 & \text{for } -1 \leqq x \leqq a \\ (x - a)^r & \text{for } \quad a \leqq x \leqq 1 \end{cases} \qquad r = 1, 2 \ldots$$

$F_1(a;\ x)$ then is the well-known series of straight broken lines. $F_r(a;\ x)$ for $r = 2, 3 \ldots$ originates if the rectilinear mean camber line is bent up behind the point $x = a$ in the form of a quadratic (cubic or higher) parabola. (Compare fig. 3.) The introduction of these mean camber lines $F_r(a;\ x)$ is advisable because - as may be noted here without proof - from them any arbitrary piecewise rational mean camber line may be obtained by superposition for various values of r and a.

In our case of a series of straight lines rounded according to figure 2, the deformation is expressed by mean camber lines of the type F_2:

$$z = \frac{C}{4f}\left[F_2(a - f;\ x) - F_2(a + f;\ x)\right]$$

The appertaining pressure distribution is therefore found easily if the pressure distribution Π belonging to the mean camber line $F_2(a;\ x)$ is known. If, as is customary, angle coordinates are introduced ($x = -\cos\Theta$, $a = -\cos\varphi$, Π is:

$$\Pi = \frac{1}{\pi}\rho v^2 e^{i\nu t}\left[(1 + T)\Pi_1 + (1 + T)\omega\Pi_2 + \Pi_3 + \omega\Pi_4 + \omega^2\Pi_5\right]$$

in which

$$\Pi_1 = \Phi_2 \cot\frac{\Theta}{2}$$

$$\Pi_2 = \Phi_{22}\cot\frac{\Theta}{2}$$

$$\Pi_3 = -2\Phi_3\cot\frac{\Theta}{2} + 4(\pi - \varphi)\sin\Theta + 4(\cos\varphi - \cos\Theta)L(\Theta,\varphi)$$

$$\Pi_4 = -\Phi_4\cot\frac{\Theta}{2} + 4(\pi - \varphi)\sin\Theta(\cos\varphi - \cos\Theta) +$$

$$2(\Phi_2 - \Phi_3)\sin\Theta + 4(\cos\varphi - \cos\Theta)^2 L\ (\Theta,\varphi)$$

$$\Pi_5 = \frac{2}{3}(\cos\varphi - \cos\Theta)^3 L(\Theta, \varphi) + \frac{2}{3}(\pi - \varphi)\sin\Theta\cos^2\Theta -$$

$$2\sin\Theta\cos\Theta\left[(\pi - \varphi)\cos\varphi + \frac{1}{3}\sin\varphi\right] +$$

$$2\sin\Theta\left[\left(\frac{1}{6} + \cos^2\varphi\right)(\pi - \varphi) + \frac{5}{6}\sin\varphi\cos\varphi\right]$$

The functions Φ_2, Φ_3, Φ_4, and $L(\Theta, \varphi)$ are known from reference 1, Φ_{22} is newly introduced. It is assumed to signify:

$$\Phi_{22} = (\pi - \varphi)\left(\frac{1}{2} + \cos\varphi + \cos^2\varphi\right) + \frac{1}{3}\sin\varphi\left(\frac{1}{2} + \cos\varphi\right)(4 + \cos\varphi)$$

3. Equivalent Systems for Wings with Control

Surfaces with Aerodynamic Balance

Several suggestions have been made for this type, for which the appertaining air forces are, to the greatest part, already tabulated. I enumerate those suggestions in the succession in which they were put forward.

First Suggestion (Küssner). - The mean camber line is rectilinearily continued from the leading edge of the wing over the slot to the control-surface axis and is broken only behind it. (Compare fig. 4.) This model is usable for those - not too frequent - profile forms where the nose of the control surface protrudes only slightly from the total profile although the control surface is deflected. Calculations for it have been performed first by the Henschel firm, later by Dietze in Forschungsbericht Nr. 1262.

Second Suggestion (Küssner). - The essential characteristic of this suggestion is that the slot is bridged by an oblique line connecting the trailing edge of the fin and the leading edge of the control surface. (Compare fig. 5.) The limiting process in which the slot width is made to approach zero is formally carried out as far as practicable: for numerical values, see reference 1. At this opportunity, an error in that report may be corrected: On page 348, top, table 3, line 1, column 3, Φ_{18} should read $-\Phi_{18}$. The numerical values of Φ_{18} are therefore negative, not positive, as indicated there. We are indebted to Mr. Dietze for bringing this to our attention.

Third Suggestion (Söhngen). - Here we operate with a discontinuous mean camber line. We arrive at it by assuming a mean camber line each for control surface and fin separated by the slot (tandem arrangement) and then going to the limit of zero slot width, compare figure 6; for numerical values, see reference 1.

Fourth Suggestion. - The mean camber line is assumed to run rectilinearly in the fin and also in the control surface behind the axis. Let a parabolic arc be inserted between control-surface axis a and leading edge of the control surface k which at a is tangent to the rectilinear part of the mean camber line of the control surface. (Compare fig. 7.) The amplitude of the control surface is assumed to be C. This mean camber line then is expressed by the functions $F_r(a;x)$ introduced above as follows:

$$z = -C\left[F_1(k;\ x) + \frac{1}{a - k}\left\{F_2(a;\ x) - F_2(k;\ x)\right\}\right]$$

The appertaining pressure distribution is calculated from the known pressure distributions of F_1 and F_2.

4. Experimental Mean Camber Line

All suggestions which have become known so far amount to replacing the profile by a mean camber line which, on one hand, renders the profile and its downwash as well as possible, and is, on the other, of the simplest possible form. The arbitrariness inherent to all considerations concerning correct selection of the mean camber line so far reveals itself in the inaccurate mode of expression above.

Now, we will proceed in reverse. Conclusions are to be drawn from the pressure distribution - which we assume as experimentally prescribed - to the mean camber line and downwash, respectively, on which it is based. The required apparatus of formulas is developed below. Possibly, certain empirical rules for the selection of mean camber lines may be gained by such an evaluation of measurements.

If, besides the designations already used, ω_r is introduced for the real reduced frequency, there results for the downwash:

$$w(x,\ t) = \frac{\omega_r}{2\pi\rho v}\oint_{-1}^{1} \Pi(\xi,\ t)g(\omega_r(x-\xi))\ d\xi$$

therein:

$$g(z) = \frac{1}{z} - ie^{-iz} \oint_{-\infty}^{z} \frac{e^{it}}{t} dt$$

or

$$\text{Re } g(z) = \frac{1}{z} - \sin z \text{ Ci } |z| + \cos z \left(\frac{\pi}{2} + \text{Si } z\right)$$

$$\text{Im } g(z) = - \cos z \text{ Ci } |z| - \sin z \left(\frac{\pi}{2} + \text{Si } z\right)$$

Values of these functions accurate to four digits are calculated and indicated in a planned research report. The values for

$$\frac{1}{2\pi} \text{ Re } g(z) \text{ and } \frac{1}{2\pi} \text{ Im } g(z)$$

appear in a report by Possio (reference 2) under the designation $-\frac{v_f}{a\omega}$ and $-\frac{v_q}{a\omega}$ for the Mach number $\lambda = 0$.

Since the deformation z and the downwash w are in the linear differential relation

$$w = \frac{\partial z}{\partial t} + v \frac{\partial z}{\partial x}$$

z is not unequivocally determined by w. This is clarified in the simplest manner by considering the "zero deformation", that is, such deformations as result in the downwash 0. They are, as far as they are harmonic:

$$z = Cve^{i\nu t} e^{-\omega x} = Cve^{-\omega(x-vt)}$$

They may be visualized in a simple manner. The curve $z = |Cv| \sin (\omega_r x)$, thus a sine curve with the period $\frac{2\pi}{\omega_r}$, is drawn on a film strip and the latter is moved along the x axis at the free-stream velocity v. One now observes that part of the sine line which is visible at the location of the wing, that is, from $x = -1$ to $x = 1$. If one lets the wing deform in such a manner that its mean camber line always coincides with this section of the sine line, it obeys the law of motion just indicated which guarantees always vanishing downwash. Such a deformation may be superimposed on any arbitrary profile deformation without any change in the downwash and, therewith, in the aerodynamic reactions.

Except for the addition of such a "zero deformation" the mean camber line then results from the pressure distribution as follows:

$$z(x,\ t) = \frac{1}{2\pi\rho v^2}\int_{-1}^{1} \Pi\ (\xi,t) f\left(\omega_r\ (x - \xi)\right) d\xi$$

therein

$$f(z) = e^{-iz}(1 - iz) \oint_{-\infty}^{z} \frac{e^{it}}{t}\, dt + 1$$

or

$$\operatorname{Re} f(z) = (\cos z - z \sin z)\operatorname{Ci} |z| + (\sin z + z \cos z)\left(\frac{\pi}{2} + \operatorname{Si} z\right) + 1$$

$$\operatorname{Im} f(z) = -(\sin z + z \cos z)\operatorname{Ci} |z| + (\cos z - z \sin z)\left(\frac{\pi}{2} + \operatorname{Si} z\right)$$

The values of these two functions also are calculated within four digits.

5. Obtaining the Unsteady Pressure

Distribution from the Steady One

In view of the difficulties impeding theoretical comprehension of the air forces which, for instance, still oppose the experimental determination of unsteady pressures, one should not reject the idea of a possible utilization of steady measurements in the wind tunnel for the unsteady problem by means of a theoretical method. For this purpose, we shall transform the theoretical result, as it was analyzed at the beginning of this representation, in such a manner that in the final formula the mean camber line does not appear at all or is only little effective. It is true that, for its derivation, it must be assumed for certain that an oscillating mean camber line may be adopted for the oscillating profile. For simplicity, we shall at first presuppose the latter to be continuous.

Let $z(x,t) = \bar{z}(x)e^{i\nu t}$ signify the oscillating mean camber line, thus $\bar{z}(x)$ the mean camber line in maximum deflection. $\Pi(x,\ t)$ will be the unsteady pressure distribution of the oscillating mean camber line $z(x,t)$, $\bar{\Pi}(x)$ the steady pressure distribution of the mean camber line in the maximun deflection $\bar{z}(x)$. According to the integral representation for the unsteady pressure distribution:

$$\Pi(x,t) = \frac{2}{\pi}\rho v^2 e^{i\nu t}\oint_{-1}^{1}\left[\omega\bar{z}(\xi) + \bar{z}'(\xi)\right]K(x,\xi)d\xi$$

where the kernel $K(x,\xi)$ is a well-known function. On the other hand, the mean camber line $\bar{z}(x)$ may be expressed by the steady pressure distribution connected with it:

$$\bar{z}'(x) = \frac{1}{2\pi\rho v^2}\oint_{-1}^{1}\Pi(\xi)\frac{d\xi}{x - \xi}$$

If this is introduced into the formula for $\Pi(x,t)$ the mean camber line $\bar{z}(x)$ is eliminated and replaced by the steady pressure distribution $\Pi(x)$ and a few functions and constants derived from it, $A(x)$, $M(x)$, c_1, c_2, c_3:

$$\rho v^2 A(x) = \int_x^1 \Pi(x)\, dx$$

$$\rho v^2 M(x) = \int_x^1 x\Pi(x)\, dx$$

$$\rho v^2 c_1 = \frac{1}{\pi}\int_{-1}^{1}\Pi(x)\, dx$$

$$\rho v^2 c_2 = \frac{1}{\pi}\int_{-1}^{1}x\Pi(x)\, dx$$

$$\rho v^2 c_3 = \frac{1}{\pi}\int_{-1}^{1}\Pi(x)\ln\left(2\sqrt{1 - x^2}\right) dx$$

To within factors, $A(x)$ is the lift and $M(x)$ the moment of the profile section from the point x to the trailing edge, the latter referred to the wing chord center. However, $M(x)$ will appear only in the combination $M(x) - xA(x)$, which signifies the same moment, referred to the point x itself. To within factors, c_1 and c_2 are lift and moment coefficients. For c_3, we do not have physical interpretation.

As the result of the described calculation (in which, besides, at one point the presupposition of the constant mean camber line must be used), we obtain the following new formula for the unsteady pressure disbribution:

$$\begin{aligned}\frac{1}{\rho v^2 e^{i\nu t}}\Pi(x,t) = {} & (1+T)\frac{1}{2}c_1\sqrt{\frac{1-x}{1+x}} + \\ & \omega(1+T)\frac{1}{2}\left[\bar{z}(1)+\bar{z}(-1)-c_3-c_2\right]\sqrt{\frac{1-x}{1+x}} + \\ & \left[\frac{1}{\rho v^2}\Pi(x) - c_1\sqrt{\frac{1-x}{1+x}}\right] + \\ & \omega\left[c_2\sqrt{\frac{1-x}{1+x}} + 2c_1\left(\frac{\pi}{2}-\arcsin x\right) - 2A(x)\right] + \\ & \omega^2\left[\left\{\bar{z}(1)+\bar{z}(-1)-c_3-c_1\right\}\sqrt{1-x^2} + \right. \\ & \left.(c_1x-c_2)\left(\frac{\pi}{2}-\arcsin x\right) + M(x) - xA(x)\right]\end{aligned}$$

The correctness of this formula was checked for the known cases of bending (vertical displacement) oscillations, of angle and of control-angle oscillation. The bending oscillation will be briefly discussed because this will help in understanding the formula.

For the latter $\Pi(x) = 0$ because the mean camber line is a section extended in the direction of the approach flow. Consequently, c_1, c_2, c_3 as well as the functions $A(x)$ and $M(x)$ are zero, and $\Pi(x,t)$ is reduced to the terms containing $\frac{1}{2}\left[\bar{z}(1) + \bar{z}(-1)\right]$ which contribute exactly the pressure distribution of a bending oscillation.

These terms in our formula are of interest because they still represent, as it were, a rudiment of the otherwise eliminated mean camber line. Their appearance, however, is easily understandable: If one superimposes a bending oscillation on an arbitrary oscillation, $\Pi(x)$ and the functions and constants formed from it A, M, c_1, c_2, c_3 remain unchanged. Without the terms with $\frac{1}{2}\left[\bar{z}(1) + \bar{z}(-1)\right]$ the pressure distribution of an arbitrary oscillation would therefore not change due to the addition of a bending oscillation; they, precisely, cause the pressure distribution of the additional bending oscillation also to be contributed.

Interpretation on the Profile. - Whereas, so far all considerations were made within the scope of linearized theory, our result now is to be interpreted also for the case of a thick profile. This is possible because the mean camber line no longer enters into the final formula, although with an exception - the quantities $\bar{z}(1)$ and $\bar{z}(-1)$. These, however, are no longer dependent on the shape of the mean camber line and may be taken from the profile without assumption of a mean camber line, since they are nothing else but the amplitudes of leading and trailing edge, respectively, of the wing. Thus, the problem is merely: What is the significance of the "pressure distribution" $\Pi(x)$ on the profile?

In order to make our deliberations more illustrative, we shall visualize below a profile with oscillating control surface. The control surface deflection will be denoted by α; generally α will be called an oscillation or deformation parameter.

Moreover, a preliminary remark: "Pressure" will, as is customary in linear theory, always signify the difference between the pressures on upper and lower side of the profile.

If a profile is prescribed, the steady flow which sets in when the profile is at rest also is superimposed on the oscillation process. The steady flow, however, is not contained in theory by the oscillating mean camber line. This is concluded, for instance, from the fact that in the case of the deflection $\alpha = 0$ the mean camber line is a straight line and contributes steady zero pressure. Or let us visualize the oscillating S-shaped mean camber line mentioned before. According to the principle of linear superposition $\Pi(x)$ is, at any rate, the difference between the steady pressure distribution $\Pi_{\alpha_{max}}(x)$ - which corresponds to the maximum deflection α_{max} - and $\Pi_0(x)$ which appears for the state of rest $\alpha = 0$

$$\Pi(x) = \Pi_{\alpha_{max}}(x) - \Pi_0(x)$$

$\Pi_\alpha(x)$ is to be, at least in theory, a linear function of α so that we also may write

$$\Pi(x) = \alpha_{max}\left(\frac{\partial \Pi_\alpha(x)}{\partial \alpha}\right)_{\alpha = 0}$$

that is, $\Pi(x)$ is, except for a constant factor, the partial derivative of the pressure distribution of the profile according to the deformation parameter. Correspondingly, we obtain, except for constant factors, $A(x)$ and $M(x)$ from the lift and the moment for the profile section from x to the trailing edge if we differentiate partially according to the deformation parameter α. Analogously, there results for c_1 and c_2

$$c_1 = \frac{\alpha_{max}}{\pi} \frac{\partial c_a}{\partial \alpha}$$

$$c_2 = \frac{2\alpha_{max}}{\pi} \frac{\partial c_m}{\partial \alpha}$$

with c_a and c_m signifying the customary lift and moment coefficients, the latter referred to the wing chord center as point of reference. For c_3 there is no more illustrative interpretation than the one given by the qualifying equation itself.

Thus the following method is advisable for obtaining $\Pi(x)$:

The profile is provided with pressure orifices which, as far as possible, are opposite one another; the pressures at the orifices are measured for several angles of attack α of the control surfaces, for instance, for $\alpha_0 = 0$ and for two positive angles of attack α_1, α_2, and two negative ones α_3, α_4. If one then forms the difference between upper and lower side for the pair of orifices at the point x, the pressures $\Pi_\alpha(x)$ for the various control-surface deflections $\alpha = \alpha_0 \ldots \alpha_4$ result. One then plots for each pair of measuring stations $x\Pi_\alpha(x)$ over α and connects the five points obtained by a curve which in linear theory should be a straight line. Through the direction of a faired curve or, better, of the tangent for $\alpha = 0$ one then obtains $\frac{\partial \Pi\alpha(x)}{\partial \alpha}$, thus after multiplication by α_{max} the desired quantity $\Pi(x)$. From this function of x one obtains by integration $A(x)$, $M(x)$, c_1, c_2, c_3 and finally, by application of the above formula, the unsteady pressure distribution $\Pi(x,t)$.

To sum up: In order to avoid the arbitrary fixing of a certain mean camber line, which of itself is necessary for every application of the theory, the result of the theory was transformed in such a manner that instead of the mean camber line only quantities will enter into the final formula which can be simply interpreted on the profile. Aside from the amplitudes of profile nose and trailing edge, they all are derived from the differential quotient of the steady pressure distribution according to the deformation parameter. This function along the wing chord may be determined according to the above method by evaluation of wind-tunnel tests. Thereby, it becomes possible to perform wind-tunnel tests rather than to fix to a certain degree arbitrarily a mean camber line.

Our considerations so far are based on a continuous mean camber line. For discontinuous mean camber lines which correspond to the limiting case of a tandem arrangement for zero slot width, our formula for $\Pi(x, t)$ can be easily modified:[1] If a is such a point of discontinuity so that $\bar{z}(a - 0) \neq \bar{z}(a + 0)$, one has to add on the right side the

$$\text{additional term} = \left[\bar{z}(a + 0) - \bar{z}(a - 0)\right] \left\{\Pi_{step}(a; x,t) - \frac{1}{2}\Pi_{bending}(x,t)\right\}$$

where $\Pi_{bending}(x,t)$ signifies the pressure distribution of the bending oscillation, whereas $\Pi_{step}(a;x,t)$ appertains to the oscillation of the control surface step (with open step for $x = a$). Thus

$$\Pi_{bending}(x,t) = \rho v^2 e^{i\nu t}\left\{(1 + T)\omega\sqrt{\frac{1 - x}{1 + x}} + 2\omega^2\sqrt{1 - x^2}\right\}$$

$$\Pi_{step}(a;x,t) = \rho v^2 e^{i\nu t}\frac{1}{\pi}\left\{(1 + T)\omega\sqrt{\frac{1 - x}{1 + x}}\left[\frac{\pi}{2} - \text{arc sin } a + \sqrt{1 - a^2}\right] + \omega\left[2\Lambda(x,a) - 2\sqrt{1 - a^2}\sqrt{\frac{1 - x}{1 + x}}\right] + \omega^2\left[2\left(\frac{\pi}{2} - \text{arc sin } a\right)\sqrt{1 - x^2} + 2(x - a)\Lambda(x,a)\right]\right\}$$

The decision as to whether to adopt a continuous or discontinuous mean camber line must be made at the outset. The method described above does not give information on that question.

Since the parameters of our formula are taken from the experiment, infinite pressure points no longer appear in the pressure distribution

[1]The following three formulas have been added only for the printing.

except for the leading edge where $\sqrt{\frac{1 - x}{1 + x}}$ becomes infinite. Since infinite pressure points are better avoided, the function $\sqrt{\frac{1 - x}{1 + x}}$ could be replaced by the derivative of the pressure distribution of the profile with respect to the angle of attack of the total profile since the latter, as is well known, is the pressure distribution of a flat plate at an angle of attack. However, this suggestion only has the character of an additional hypothesis.

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2. Possio, C.: L'Azione aerodinamica sul profilo oscillante in un fluido compressibile a velocità iposonora. L'Aerotecnica Bd. XVIII; 1938, pp. 441 - 458.

Figure 1.- Mean camber line with S-camber.

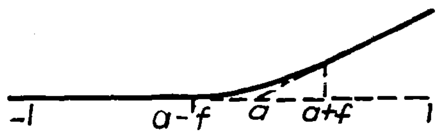

Figure 2.- Mean camber line with rounded break.

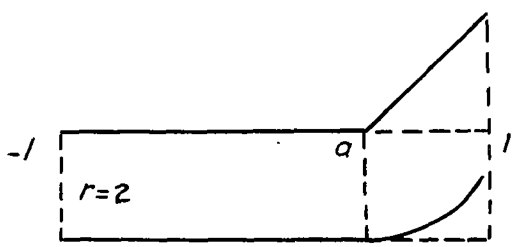

Figure 3.- Mean camber line $F_r(a;x)$ for $r = 1$ and 2.

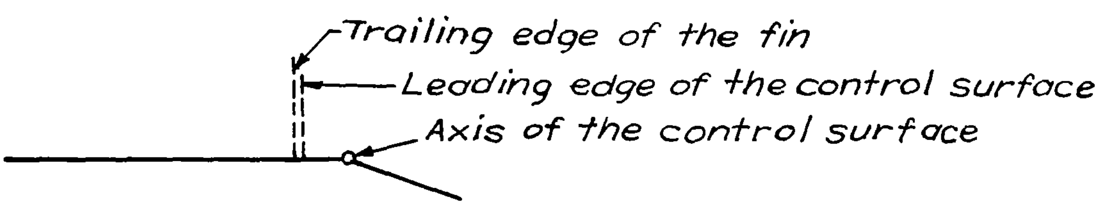

Figure 4.

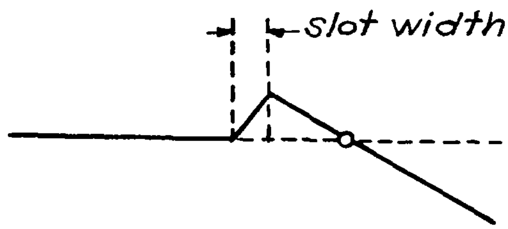

Figure 5.

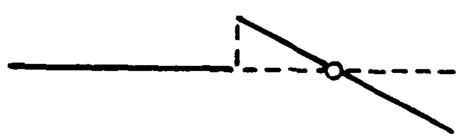

Figure 6

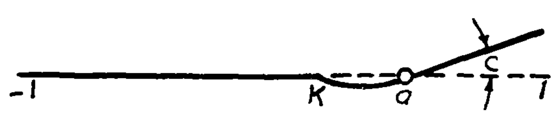

Figure 7

Discussion Following the Lecture of L. Schwarz

Groth: Concerning the problem of the measurement of unsteady pressure distribution, I should like to make the following remark:

In the high-speed wind tunnel of the LFA Braunschweig, one uses as special measuring method the interference method which permits, according to the interference principle of Mach-Zehnder, density measurements on bodies in a flow. Since light reproduces, free from inertia, any change in the flow state, the performance of pressure distribution measurements is possible also for unsteady processes. We therefore prepared as one of the first problems for research interference measurements on the oscillating wing where a wing placed between end plates fixed in space is made to oscillate in pure angle or bending oscillations. The interference photographs then show the pressure distribution existing in the separate states of the oscillating motion and permit statements on the development of the boundary layer on the profile. The first measuring results may probably be expected at the end of this year. On the occasion of the first testing of the interference method in a small provisory wind tunnel, we were able to make sure that reliable interference photographs on the oscillating wing are possible.

Leiss: In adapting a mean camber line to steady measurements, one has to pay attention to the fact that slot and separation influences may have entirely different effects for steady and for unsteady conditions. According to deliberations discussed occasionally by other investigators, an adjustment to steady behavior might lead to a mean camber line which, for unsteady conditions, might stress too heavily for instance the slot influence.

Küssner: The vortices at the slot of the oscillating wing will also appear for small ω values; we do not know what happens in case of large ω values. Of course, we shall make measurements and then compare these measurements with the theory.

Stender: I made before, elsewhere, the suggestion that the influence of a sudden deformation of the tail unit profile by control-surface deflection on the pressure variation at different points of the tail surfaces for high Mach numbers be investigated. An actuation of the control surfaces probably takes effect in the region of the tail unit nose only much later.

I believe that one must also consider the flow about the trailing edge appearing for every control-surface deflection because it has a phase angle relative to the control-surface deflection. Perhaps one may, as an expedient, make the assumption of a slight periodical bending of the trailing edge.

Translated by Mary L. Mahler
National Advisory Committee
for Aeronautics

III. COMPARATIVE CALCULATIONS CONCERNING AERODYNAMIC BALANCE OF CONTROL SURFACES

By F. Dietze

I. Preliminary Remarks

A number of approximation formulations have been developed in order to include aerodynamic balances of control surfaces in flutter calculations; these formulations are based on widely different ideas - regarding consideration of the velocity distribution of the oscillating wing in the forward part of the control surface. At present, a reliable answer to the question of how far these various interpretations correspond to actual conditions in the individual case is not possible, due to the lack of suitable measuring data on unsteady air forces and air-force distributions. Thus the present investigations are, chiefly, restricted to a comparison between the various concepts, with respect to their effect on the air forces and on the critical velocity. In order to obtain at least a certain criterion regarding the reliability of the different concepts, an example for flutter of a wing with aerodynamic control-surface balance (investigated in the wind tunnel by Voigt) was checked by calculation according to the different concepts.

II. Characterization of the Concepts

Figure 1, right, shows sketches of the profile mean camber lines on which the individual concepts regarding consideration of the velocity distribution in case of fixed fin and rotating control surface are based. On the left a few control-surface structures are indicated in order to illustrate how the separate concepts could be coordinated to actual construction types, merely on the basis of design and mounting of the control surfaces. The case of the control surface without aerodynamic balance has been included only for comparison.

Individually, the concepts are characterized as follows:

<u>Concept a (older concept of Küssner)</u>: The break of the wing is assumed to lie within the control-surface region. Thus the normal velocity components are included, according to the actual kinematic boundary conditions, only in the fin part and in the control-surface part behind the break of the control surface; in the control-surface part in

front of the break (of the control surface) the normal velocity component is taken into consideration according to the kinematic conditions of the fin.

Concept b: Here the model for the velocity distribution shows an oblique step in the front part of the control surface, so that the normal velocity components near the leading edge of the control surface are included only partly according to the kinematic boundary conditions. The rearward point of break (tip of the step) should lie - whenever possible - at such distance behind the leading edge of the control surface that the slope of the step is of about the same order of magnitude as the control-surface angle of rotation.

Concept c (Söhngen): The normal velocity components are here fully included according to the kinematic boundary conditions. As Söhngen has shown, the flow through the slot contributes for steady conditions to the total lift; these contributions are small of higher order. Since it is assumed that this influence takes rather even less effect for unsteady conditions, it is here also neglected.

Concept d (Küssner-Schwarz): This concept is characterized by a mean camber line with closed vertical step. Terms resulting in infinitely large amounts appear in the air-force law. According to a suggestion by Küssner, these infinitely large amounts are to be replaced by finite amounts from comparative steady calculations.

So much for the present for characterization of the individual concepts. It is, of course, perfectly feasible to vary or to combine these concepts. A variation may, for instance, be made by replacing (in the design with oblique step) the rectilinear connecting piece between fin and control surface by a connection with several breaks or by closing the transition from fin to control surface by a smooth curve (without break).

A few parameters for the separate concepts are still open, for instance, position of the point of break, step width, etc. These parameters may be determined by bringing on conformity with appropriate measuring values in the air forces. Following, such a possibility will be discussed in more detail.

III. Effect on the Air Forces

The comparison between the individual concepts with respect to their effect on the air forces is performed for the air-force distribution over the wing chord and for the control-surface moment, both for fixed fin and harmonically oscillating control surface. The control-surface chord ratio is for all concepts $\tau = 0.2$; the control-surface axis of rotation which simultaneously is the axis of reference for the control-surface moment lies at one-fourth of the control-surface chord behind the leading edge of the control surface.

The parameters still disposable in the various concepts were fixed as follows:

Concept	Parameter at disposal
Break edge shifted rearward	Break edge in the control surface axis of rotation
Oblique step	Step width = 4 percent of the control surface chord
Open step	Point of rotation of the mean camber line of the control surface in the control-surface axis of rotation
Closed vertical step	

a. Air-force distribution

In figure 2 the air-force distribution due to control-surface rotation is plotted against the wing chord for the different concepts. The figure at the left shows, by way of comparison, the steady distribution, the two at the right show the unsteady distributions.

All interpretations result in about the same air-force distribution with respect to direction; only for the interpretations with oblique and vertical closed step narrow pressure peaks occur additionally which are, however, without significance for the total lift. In the last named two interpretations the pronounced reduction in the fin region of the air-force part that is in phase with the motion is particularly conspicuous. For steady conditions and for the part in phase with the motion, the pressure distribution of the concept with open step does not differ at all from that of the unbalanced control surface.

b. Air moment on the control surface

In figure 3 the control-surface moment due to control-surface rotation, for the various interpretations, is plotted against the reduced frequency. The control-surface moment therein is referred to the control-surface axis of rotation.

The reduction of the control-surface moment is composed chiefly of two parts:

1. The control-surface moment is reduced, because the axis of reference of the moment, the control-surface axis of rotation, is shifted rearward. The reduction here is expressed in the distance of the curves of the unbalanced control surface, for instance from the curves appertaining to the concepts with open step: for the part in phase with the motion as well as for the part which is out of phase with respect to the motion by $\Pi/2$.

2. The other component stems from the different consideration of the velocity distribution in the forward part of the control surface for the individual interpretations and is expressed in the mutual distances of the curves of the balanced control surface. It is noteworthy that this influence chiefly takes effect only in the component which is in phase with the motion, and that the differences between the separate interpretations are almost independent of the frequency. This fact yields, for practical application, the following rule for mathematical representation of the aerodynamic control-surface balance.

As long as no reliable, unsteady air-force measurements exist, a selection among the different concepts concerning the different control-surface forms with aerodynamic balances can be made - at least approximately - solely on the basis of the control-surface degree of balance for steady conditions.

Figure 4 shows an example of how such a selection may take place. The steady moment coefficients $dc_{mR}/d\gamma_R$ of the control surface due to control surface deflection are here compiled for various forms of nose development of the control surface. The value $dc_{mR}/d\gamma_R = 0.01$ corresponds to the unbalanced control surface. The measured results were taken from the report Fo 187 entitled "Momentenmessungen im Windkanal an 22 verschiedenen Rudern" (Moment Measurements in the Wind Tunnel on 22 Different Control Surfaces) in agreement with the Hamburg Flugzeugbau Blohm & Voss, who performed the measurements. The interpretations with break shifted rearward, with oblique step and with open step were coördinated to the separate control surfaces in such a manner that the individual interpretations result in the same degree of balance as the control surfaces.

In first approximation, one may derive from this investigation roughly perhaps the following rules for use in a selection among the individual interpretations:

1. The interpretation with rearward shifted break will always be preferable when - as is the case for the first two control surfaces - the control surface nose does not protrude from the contour line of the profile.

2. The interpretation with open step will always be preferable when - as is the case for the last two control surfaces - the control-surface nose protrudes even for small control-surface deflections.

3. A medium type of construction is effectively included in the concept with oblique step. One has only to be careful not to let the step become too steep. In a dubious case, the concept with open step is preferable.

IV. Effect on the Result of a Flutter Calculation

a. Variation of the position of the control-surface axis of rotation

Figure 5 shows the critical velocity (determined by calculation) of a model which can flutter with the two degrees of freedom - wing rotation and control-surface rotation in the sense of the two-dimensional problem - plotted against the position of the control-surface axis of rotation. It was not known what degrees of balance existed at the control surface of the model for the separate positions of the axis of rotation.

In the diagram, two further test points are plotted which only are to show that the calculation result makes sense. Figure 6 shows a systematic comparison of calculation and test.

The values for the dimensions as well as the mass and stiffness values of the model investigated are compiled in table I.

For the parameters at disposal in the individual interpretations, the same numerical values were selected as in the previous investigation.

The first remarkable fact in figure 5 is that all interpretations render the influence of the balancing effect on the critical velocity in the same sense - in this particular instance in the sense that with increasing degree of balance the critical velocity increases. If the balancing effect is overestimated, as obviously happens here in the interpretations with open and closed vertical step, the calculation - as shown in the present example - may arrive at freedom from flutter where actually the system still is subject to flutter.

TABLE I. - DIMENSIONS AND CHARACTERISTIC VALUES

OF THE INVESTIGATED MODEL

Wing chord	$l = 0.60$ m
Control-surface chord	$l_R = 0.12$ m
Wing width	$b = 1.80$ m
Wing weight	$G_{Fl} = 5.04$ kg
Mass moment of inertia of the wing (neutral axis)	$\Theta_{Fl} = 0.0544$ kg/m/s^2
Rearward position of the centroidal axis behind the forward neutral axis	$s_{Fl} = 0.011$ m
Rearward position of the elastic axis behind the forward neutral axis	$e_{Fl} = 0$ m
Control-surface weight	$G_R = 0.727$ kg
Mass moment of inertia of control surface . . (axis of rotation)	$\Theta_R = 25.2 \times 10^{-5}$ kg/m/s^2
Angular frequency of the bending oscillation	$\omega_B = 4.8$ s^{-1}
Angular frequency of the torsional oscillation	$\omega_D = 8$ s^{-1}
Angular frequency of the torsional oscillation of the control surface	$\omega_R = 0.474$ s^{-1}

b. Variation of the position of the centroidal

axis of the control surface

Figure 6 shows the critical velocity (determined by test) for a model which can perform bending motions (vertical displacement), torsional motions, and torsional motions of the control surface in the sense of the two-dimensional problem, plotted against the position of the centroidal axis of the control surface. (Concerning mass and stiffness values of compilation in IV(a).) The diagram at the left represents the critical velocity for the control surface without aerodynamic balance.

The control surface of the model is designed so that, for sufficiently small deflections, the leading edge does not yet protrude; for larger deflections a step with slot originates between fin and control surface. An absolute neglect of the control-surface rotation as is practiced in the interpretation with rearward shifted break could not render full justice to the balancing effect; on the other hand, a complete consideration of the control-surface rotation - which forms the basis for the interpretations with open step - would emphasis the balancing effect too strongly. A correct evaluation of the balancing effect will, therefore, probably lie in between these two concepts.

The calculation with the two concepts in the rearward shifted break and with open step confirms this prediction (cf. fig. 6, right). Since the calculation on the control surface without aerodynamic balance at any rate yields smaller critical velocities than the measurement, it may well be assumed that the lower curve - which thus appertains to the concept with rearward shifted break - renders the phenomenon more correctly. Aside from this, one will perfer to use for this example, the concept with rearward shifted break edge, if only for the reason that the results then lie on the safe side.

I hope to have shown in these expositions that even now, when appropriate measured data on unsteady air forces on the control surfaces with aerodynamic balance do not yet exist, one is in possession of a certain criterion for a sensible device among the different interpretations. Of course, it remains the last goal to obtain, from air-force measurements on the oscillating wing, reliable data also for the air forces on the control surface with aerodynamic balance.

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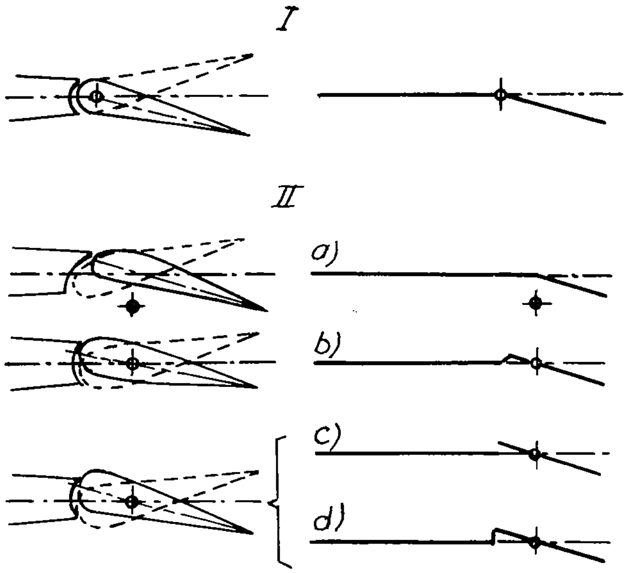

Figure 1.- Interpretations for the aerodynamic control-surface balance.
I. Control surface without aerodynamic balance.
II. Control surfaces with aerodynamic balance.

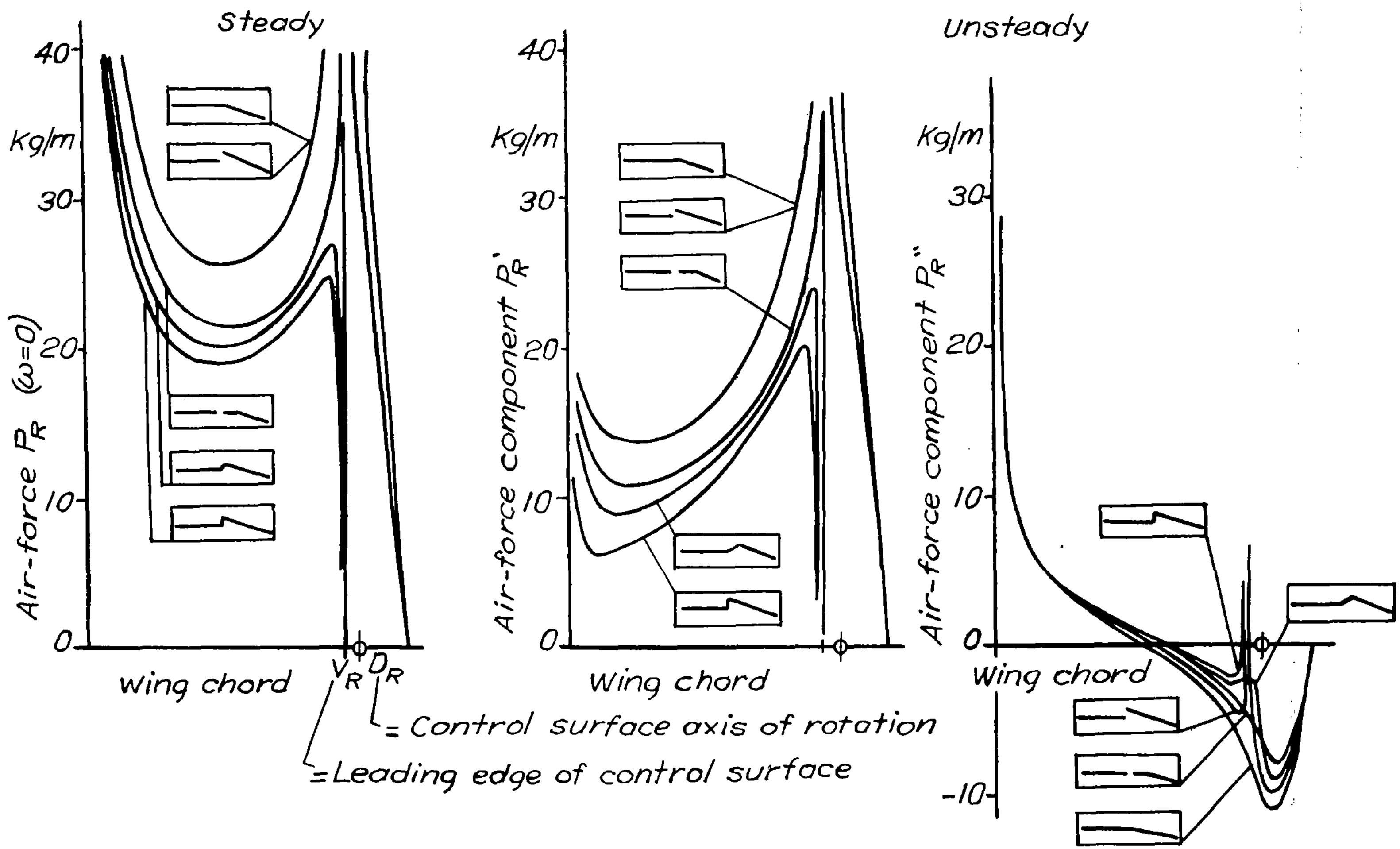

Figure 2.- Air-force distribution $p_R(t) = p_R' \cos \omega t - p_R'' \sin \omega t$ due to control-surface rotation $\gamma_R(t) = B_R \cos \omega t$, ($v$ = 20m/s, $\omega = 33s^{-1}$, $B_R = 11.5^{\circ}$).

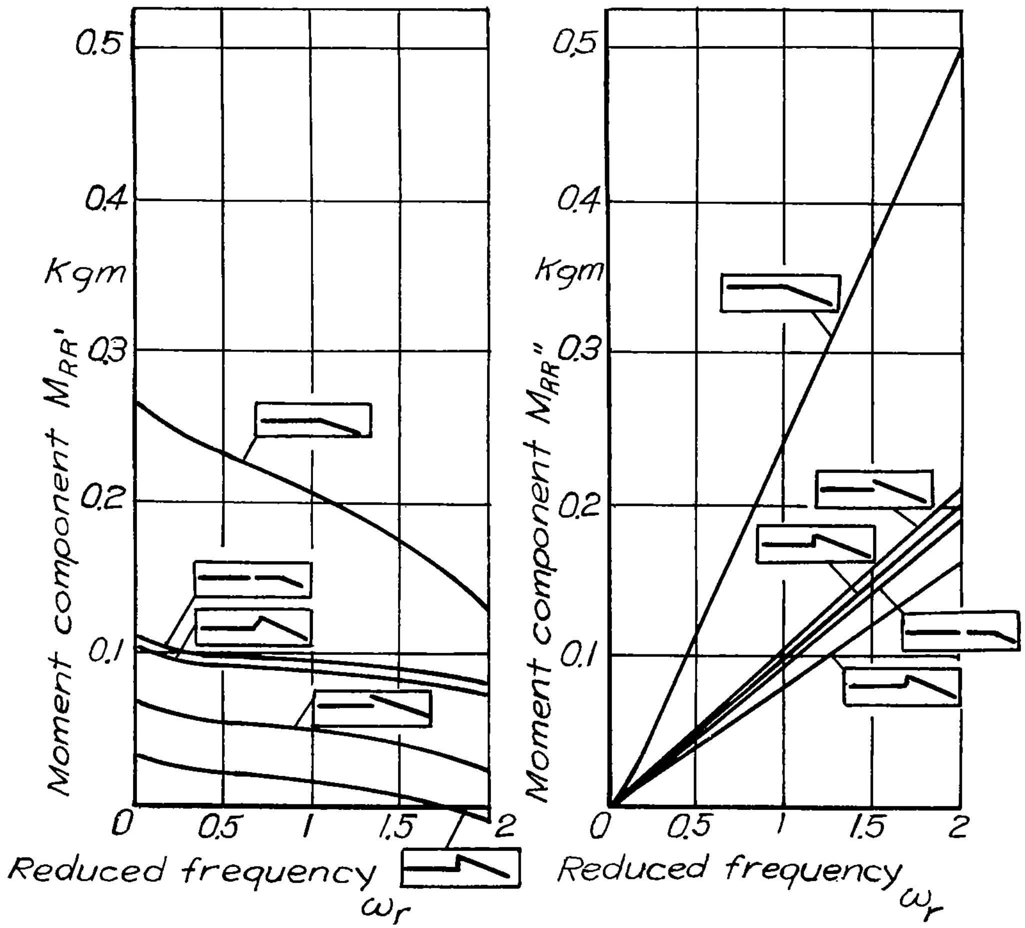

Figure 3.- Control-surface moment $M_{RR}(t) = M_{RR}' \cos \omega t - M_{RR}'' \sin \omega t$ due to control-surface rotation $\gamma_R(t) = B_R \cos \omega t$, ($v = 20\text{m/s}$, $\omega = 33\text{s}^{-1}$, $B_R = 11.5^\circ$).

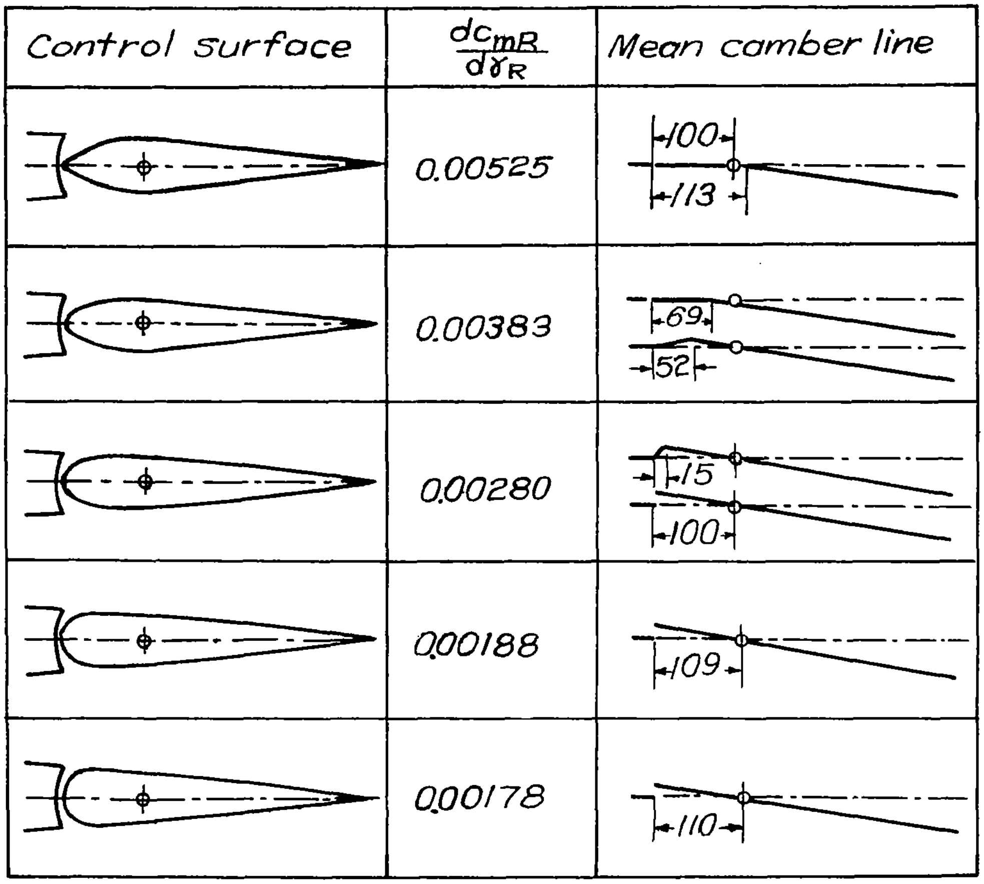

Figure 4.- Adjustment of the interpretations (numerical example).

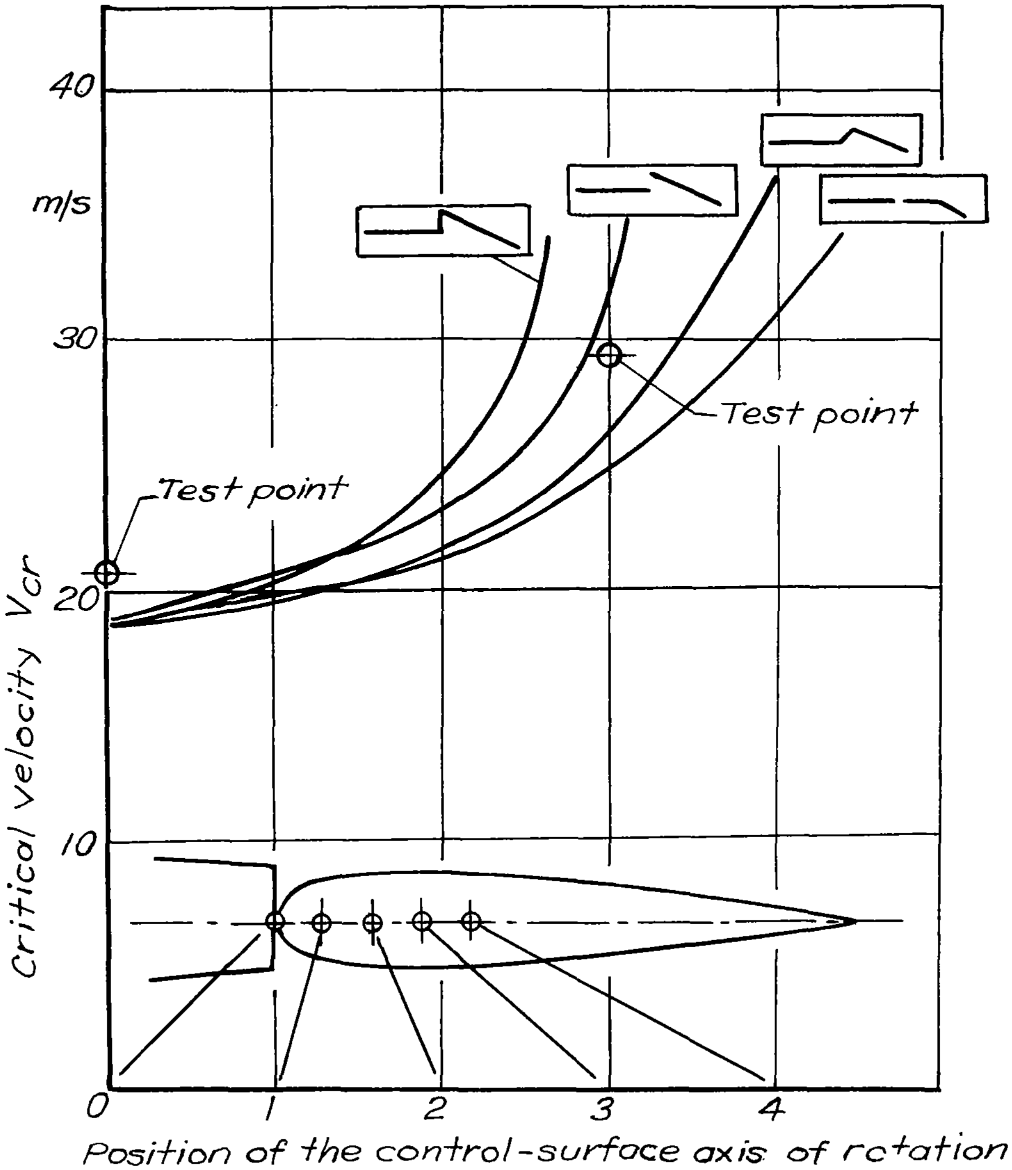

Figure 5.- Critical velocity as a function of the position of the control-surface axis of rotation.

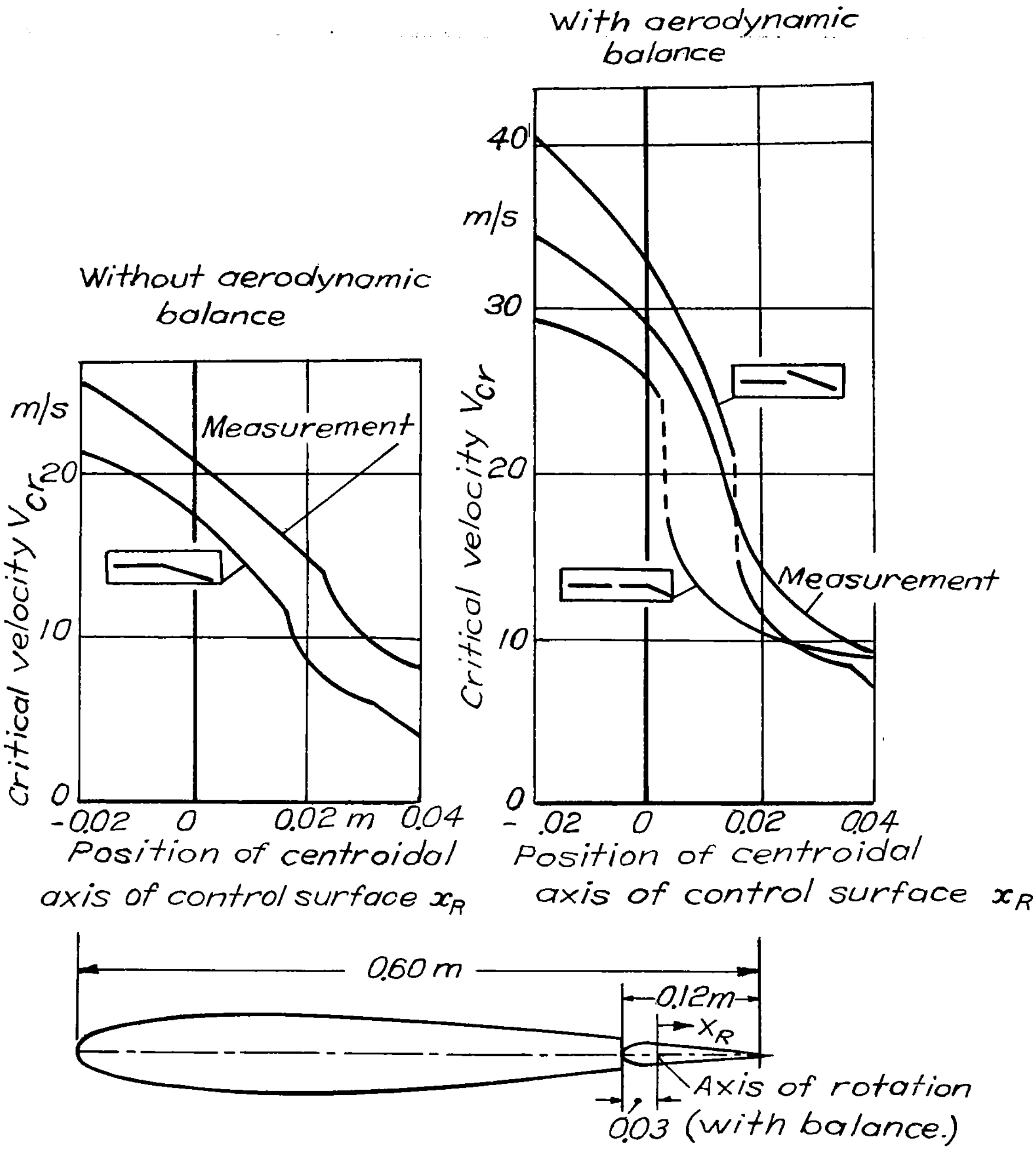

Figure 6.- Critical velocity as a function of the position of the centroidal axis of the control surface.